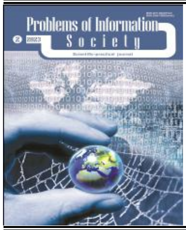


www.jpis.az

14 (2)
2023

# Comparison of signal recognition methods by combined use of appropriate evaluation criteria within the additive convolution

*Ramin R. Rzayev*[1], *Azer B. Kerimov*[2]

[1,2]Institute of Control Systems, 9A, AZ1141 Baku, Azerbaijan

[1]raminrza@yahoo.com, [2]a.k.etn00001@gmai.com

[1]*orcid.org/0000-0001-7658-2850*, [2]*orcid.org/0000-0002-1732-2474*

**ARTICLE INFO**

*http://doi.org/10.25045/jpis.v14.i2.03*

*Article history:*
Received 09 January 2023
Received in revised form
10 March 2023
Accepted 15 May 2023

**Keywords:**
Signal recognition
Euclidean metric
Evaluation criterion
Convolution
Recognition method

**ABSTRACT**

Existing signal recognition methods have both their advantages and disadvantages, which are found when recognizing signals from classes defined by different characteristic standards. Therefore, for signals from different classes, the indicators of recognition quality by one method or another can differ significantly. There is a need to create a more balanced method capable of providing the necessary stability relative to the accuracy and reliability of the final results in the process of recognizing signals from various classes. As such signal recognition method, the article proposes to use an approach based on the combine using of weighted signal proximity criteria within the additive convolution. Euclidean distances between reference points are used as evaluation criteria, which are used in the context of applying the four most well-known recognition methods: the amplitude method (the trivial Euclid method), the DDTW method using the values of the first derivatives, and methods based on the Wavelet transform and the Fourier transform.

## 1. Introduction

In (Kerimov, 2022, a), we considered some well-known signal recognition methods, the accuracy of which was compared on the basis of artificially generated families of sequentially shifted signals. It was noted that each of the analyzed recognition methods, due to its characteristic feature, answers the question of the proximity of signals in two main positions: by the amplitude characteristics of the curves reflecting the signals, or by their orientation in space, determined by the corresponding values of the derivatives of the 1st and 2nd orders. So, for example, when recognizing using the Euclidean distance, only the amplitude characteristic of the signals is used. At the same time, the DDTW (Derivative Dynamic Time Warping) method (Keogh et al., 2019) uses only the characteristic of signals associated with the spatial orientation of signal curves (values of 1st order derivatives). In particular, the 1st order derivative of the signal is used in solving many recognition problems.

In the review article (Santos et al., 2017), using the 1st order derivative, methods for recognizing signals received from spectrometers are analyzed in the context of solving biological problems. In (Liu et al., 2019) 1st order derivatives are used to correct the baseline of the signal, as well as to remove jumps in signals received from spectrometers. In (Leszek et al., 2010) using the 1st order derivative, issues related to the appearance of various noises after the sampling of the analog signal are considered. Based on the existing developments in the subject area, the importance and relevance of further research the signal recognition methods become obvious.

This article proposes to apply an approach to signal recognition based on the combined use of three of the four proximity evaluation criteria used in the above signal recognition methods within the additive convolution. To promote this idea, an artificially generated family of sequentially shifted curves is considered as recognizable signals, which we have already used in (Kerimov, 2022, b), to evaluate some known signal recognition methods for their adequacy.

## 2. Proximity evaluation criteria for comparing signals

At the preliminary stage of signal recognition, as a rule, the main features of recognition are identified and, on their basis, the corresponding distance norm is selected. Next, the recognition procedure is carried out by comparing the recognizable signals with the standard by calculating the pairwise distances between them based on the selected metric. The choice of recognition features depends on the nature of the problem being solved (the family of recognizable signals) and the applied approach. However, in all cases, the Euclidean metric is used as the basic norms of the distance between reference points of corresponding signals. So, to form the weighted additive convolution, four distance norms were chosen as evaluation criteria, using which the following known methods of signal recognition are used: the amplitude method, the method using 1st order derivative (DDTW), Wavelet and Fourier transforms. At the same time, for each of the listed methods, the corresponding features of recognition are determined.

**Amplitude recognition method (ARM).** The values of reference points are chosen as recognition features. In particular, if for two arbitrary signals $x$ and $y$ the points $a_i$ and $b_i$ ($i$ = 0, 1, …, $N$) are the reference points, respectively, then the Euclidean metric is chosen as the norm of the distance between them in the form

$$D_1(x,y)=\sqrt{\sum_{i=1}^{N}(a_i-b_i)^2}\ . \qquad (1)$$

**DDTW recognition method.** As recognition features the values of the 1st derivatives at the reference points are chosen. In the discrete case, the expression $\dot{a}(i)=[a(i)-a(i-1)]/T$ is taken as the 1st order derivative, where $a(i) = a(iT)$, $i$ = 0, 1, …, $N$; $T$ is the sampling period of the analog signal (Novozhilov, 2016). In particular, if for two arbitrary signals $x$ and $y$ the reference points are respectively the values of the 1st derivatives $p_i$ and $q_i$ ($i$ = 0, 1, …, $N$), then the Euclidean metric is chosen as the norm of the distance between them as follows

$$D_2(x,y)=\sqrt{\sum_{i=1}^{N}(p_i-q_i)^2}\ . \qquad (2)$$

**Wavelet transform (WT).** According to this recognition method, each signal is decomposed into high-frequency and low-frequency components (Saraswat et al., 2017; Song et al., 2021). Moreover, each component is characterized by the values of the so-called detailing and approximating coefficients. For example, for the signal shown in Figure 1(a), which includes 256 reference points, the WT at four levels looks like that shown in Figure 1(b). Here, average values of characteristics (coefficients) in each filtering band are chosen as recognition features. In the case under consideration (Figure 1(b)), where four decomposition levels are chosen, there are eight values of recognizable features (al-Qerem et al., 2017; Taghavirashidizadeh et al., 2022).

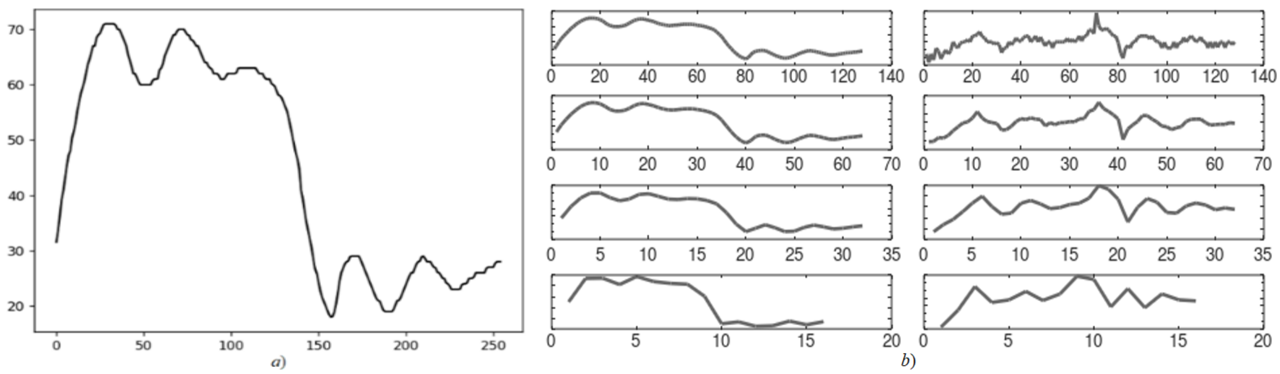


**Fig. 1.** Signal including 256 reference points (a) and its wt at 4 levels (b).

Assuming for two arbitrary signals $x$ and $y$ the average values and standard deviations of the coefficients in the high-frequency and low-frequency bands, respectively, in terms of $H_{1i}$, $L_{1i}$, $H_{2i}$, $L_{2i}$, ($i$ = 0, 1, …, $N$), where $N$ is the number of decomposition levels, the following Euclidean metric is chosen as distance norm

$$D_3(x,y)=\sqrt{\sum_{i=1}^{N}(H_{1i}-H_{2i})^2+\sum_{i=1}^{N}(L_{1i}-L_{2i})^2} \qquad (3)$$

**Fourier transform (FP).** The use of FT implies the creation of a spectral image for a recognizable signal. In particular, for the signal that includes 256 reference points (Figure 1(a)), the FT generates the corresponding amplitude spectral image (Figure 2) that includes 128 reference points. When applying the FT, the variables of the amplitude spectrum are considered as recognizable features (Hindarto et al., 2017; Ponomarev et al., 2023) which in common form for two signals are denoted as $f_{1i}$ and $f_{2i}$ ($i$ = 0, 1, …, $N$), where $N$ is the number of variables. In this case, the following Euclidean metric is chosen as the norm of the distance between two arbitrary signals $x$ and $y$

$$D_4(x,y)=\sqrt{\sum_{i=1}^{N}(f_{1i}-f_{2i})^2} \quad (4)$$

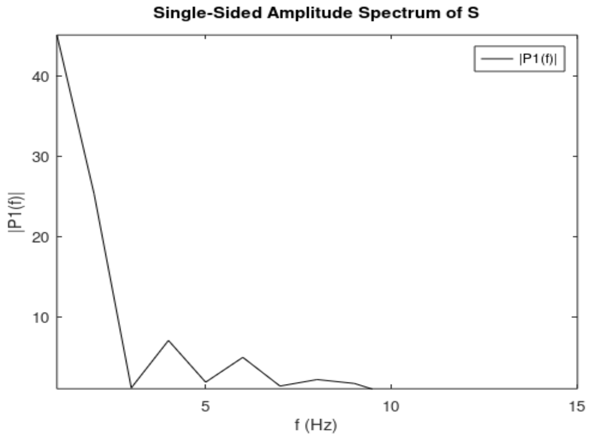


**Fig. 2.** Amplitude spectrum of the signal obtained using the ft.

Thus, in view of the foregoing, to recognize the signals the weighted additive convolution (WAC) of criteria $D_k(x, y)$ ($k$ = 1÷4) is proposed in the following form

$$D=\sum_{k=1}^{4} w_k D_k(x,y), \quad (5)$$

where $w_k$ are the weights of the evaluation criteria, reflecting the contribution of each of the above metrics (1) – (4) in solving the problem.

To identify the weights $w_k$, which a priori must satisfy the conditions: $\sum_{k=1}^{4} w_k = 1,\ 0 \le w_k \le 1$, the application of the mentioned signal recognition methods is considered on the individual basis using the example of a single class of curves. As such a class, it is chosen the artificial family of signals formed by the uniform displacement of the curves horizontally. A general analysis of recognition results is carried out on the basis of 2 criteria for assessing the adequacy of recognition method.

Criterion 1 (*sensitivity*): for a particular recognized signal, the Euclidean distances from the left standing and from the right standing signals should be approximately equal, that is, their ratio should be approximately equal to one. If the standing signals on the left and on the right are symmetrical with respect to this signal, then these distances will be absolutely equal.

Criterion 2 (*stability*): increasing the step of signal shifts cannot improve the satisfaction of recognition methods, that is, the accuracy of the recognition method must remain the same or deteriorate.

Obviously, the weights of the evaluation criteria correlate with the recognition results, that is, they are in a certain proportion with the results obtained using the specified recognition methods separately. Therefore, we have chosen the following statement as the main paradigm: how many times the recognition results will differ using the particular method, the weight coefficients corresponding to it in the additive convolution (5) will differ so many times. Based on this paradigm, the identification of weights $w_k$ is carried out on the basis of Criteria 1 and 2.

## 3. Analysis of recognition results

To form the family of recognizable signals, the signal $s_0$ was chosen as the base (standard) signal. Recognizable signals are built relative to $s_0$ by successive uniform horizontal displacement (Kerimov, 2022, b). Assuming a shift of 10 units as the step $h$, the artificial family signals $S_{10}$ = {$s_1$, $s_2$, …, $s_6$} is obtained as shown in Figure 3.

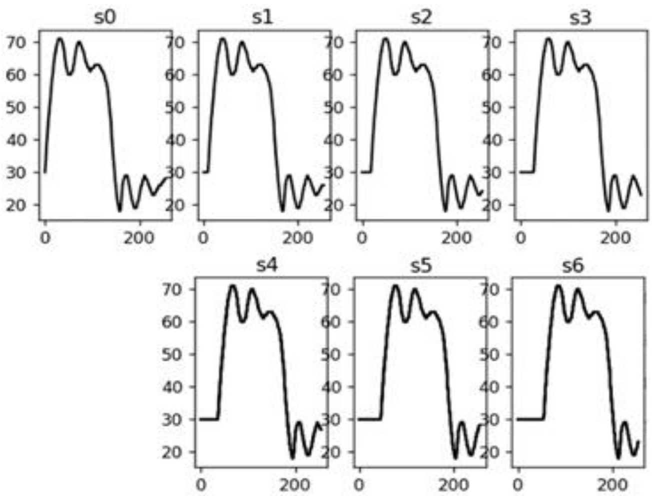


**Fig. 3.** Standard $S_0$ and family of recognizable signals $S_{10}$ = {$s_1$, $s_2$, …, $s_6$}.

For each $k$=1÷4, let us introduce the following notation: $D_k^h(x,y)$ is the distance between signals; $R_{ki}^h=\dfrac{D_k^h(s_i,s_{i+1})}{D_k^h(s_{i-1},s_i)}$ ($i=1\div5$) is the ratio between adjacent distances (that is, between the distances from the right standing ($i$+1)-th and from the left standing ($i$-1)-th signals to the $i$-th signal). Then, in these notations, the satisfaction of the methods in terms of compliance with Criteria 1 and 2 can be formulated as follows:

- the adequacy of the recognition method for compliance with Criterion 2 is evaluated based on the value of the maximum deviation, defined as $G_k^h = \max_{i=1\div5}\{1-R_{ki}^h\}$;
- the adequacy of the recognition method for compliance with Criterion 4 is evaluated based on the fulfillment of the condition $R_{ki}^{h_1} < R_{ki}^{h_2}$ ($i$ = 1÷5; $k$ = 1÷4), if $h_1<h_2$ is satisfied, where $h_1$ and $h_2$ are the steps of curve displacements (for example, horizontally) in two different families of recognizable signals.

Results of pairwise comparison of signals from the family $S_{10} = \{s_1, s_2, \dots, s_6\}$ using metrics (1) – (4) sufficiently satisfy Criterion 2. This is confirmed by the comparative estimates summarized in Tables 1 – 4, as well as subsequent calculations.

**Table 1.** Signal comparisons using the metric (1)

| | $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ |
|---|---|---|---|---|---|---|---|
| $s_0$ | 0 | 113.10 | 197.99 | 252.89 | 291.48 | 328.66 | 367.74 |
| $s_1$ | 113.10 | 0 | 112.96 | 197.67 | 252.69 | 291.45 | 328.64 |
| $s_2$ | 197.99 | 112.96 | 0 | 112.80 | 197.58 | 252.59 | 291.41 |
| $s_3$ | 252.89 | 197.67 | 112.80 | 0 | 112.66 | 197.16 | 252.39 |
| $s_4$ | 291.48 | 252.69 | 197.58 | 112.66 | 0 | 112.24 | 196.89 |
| $s_5$ | 328.66 | 291.45 | 252.59 | 197.16 | 112.24 | 0 | 112 |
| $s_6$ | 367.74 | 328.64 | 291.41 | 252.39 | 196.89 | 112 | 0 |

**Table 2.** Signal comparisons using the metric (2)

| | $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ |
|---|---|---|---|---|---|---|---|
| $s_0$ | 0 | 15.572 | 21.303 | 21.575 | 19.527 | 18.680 | 19.658 |
| $s_1$ | 15.572 | 0 | 15.490 | 21.243 | 21.461 | 19.405 | 18.609 |
| $s_2$ | 21.303 | 15.490 | 0 | 15.441 | 21.136 | 21.401 | 19.326 |
| $s_3$ | 21.575 | 21.243 | 15.441 | 0 | 15.287 | 21.046 | 21.341 |
| $s_4$ | 19.527 | 21.461 | 21.136 | 15.287 | 0 | 15.199 | 20.800 |
| $s_5$ | 18.680 | 19.405 | 21.401 | 21.046 | 15.199 | 0 | 14.925 |
| $s_6$ | 19.658 | 18.609 | 19.326 | 21.341 | 20.800 | 14.925 | 0 |

**Table 3.** Signal comparisons using the metric (3)

| | $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ |
|---|---|---|---|---|---|---|---|
| $s_0$ | 0 | 47.200 | 91.698 | 134.15 | 178.20 | 223.49 | 267.04 |
| $s_1$ | 47.200 | 0 | 45.804 | 90.048 | 135.27 | 181.64 | 226.26 |
| $s_2$ | 91.698 | 45.804 | 0 | 46.112 | 93.499 | 141.03 | 185.51 |
| $s_3$ | 134.15 | 90.048 | 46.112 | 0 | 49.772 | 98.313 | 141.90 |
| $s_4$ | 178.20 | 135.27 | 93.499 | 49.772 | 0 | 49.025 | 93.822 |
| $s_5$ | 223.49 | 181.64 | 141.03 | 98.313 | 49.025 | 0 | 47.586 |
| $s_6$ | 267.04 | 226.26 | 185.51 | 141.90 | 93.822 | 47.586 | 0 |

**Table 4.** Signal comparisons using the metric (4)

| | $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ |
|---|---|---|---|---|---|---|---|
| $s_0$ | 0 | 0.5107 | 0.9271 | 1.3881 | 1.6062 | 1.5833 | 1.6585 |
| $s_1$ | 0.5107 | 0 | 0.7558 | 1.2882 | 1.5998 | 1.5637 | 1.6182 |
| $s_2$ | 0.9271 | 0.7558 | 0 | 1.0472 | 1.4654 | 1.4747 | 1.5216 |
| $s_3$ | 1.3881 | 1.2882 | 1.0472 | 0 | 0.9194 | 1.0253 | 1.1160 |
| $s_4$ | 1.6062 | 1.5998 | 1.4654 | 0.9194 | 0 | 0.4603 | 0.7921 |
| $s_5$ | 1.5833 | 1.5637 | 1.4747 | 1.0253 | 0.4603 | 0 | 0.7124 |
| $s_6$ | 1.6585 | 1.6182 | 1.5216 | 1.1160 | 0.7921 | 0.7124 | 0 |

Tables 5 and 6 present the values of the ratios of adjacent distances $R_{ki}^{10}$ and $R_{ki}^{20}$, as well as the values of the maximum deviations $G_k^{10}$ and $G_k^{20}$ for two basic families of curves $S_{10}$ and $S_{20}$, constructed, respectively, by uniformly shifting the curves horizontally by step $h_1 = 10$ and step $h_2 = 20$.

**Table 5.** Indicators of fulfillment of criteria 1 and 2 based on the family $S_{10}$.

| Method | $k$ | $R_{k1}^{10}$ | $R_{k2}^{10}$ | $R_{k3}^{10}$ | $R_{k4}^{10}$ | $R_{k5}^{10}$ | $G_{k}^{10}$ |
|---|---|---|---|---|---|---|---|
| ARM | 1 | 0.99883 | 0.99851 | 0.99878 | 0.99625 | 0.99789 | 0.00375 |
| DDTW | 2 | 0.99611 | 0.99743 | 0.99162 | 0.99650 | 0.98405 | 0.01595 |
| WT | 3 | 0.97043 | 1.00670 | 1.07940 | 0.98498 | 0.97066 | 0.07939 |
| FT | 4 | 1.47990 | 1.38550 | 0.87795 | 0.50063 | 1.54770 | 0.54766 |

**Table 6.** Indicators of fulfillment of criteria 1 and 2 based on the family $S_{20}$.

| Method | $k$ | $R_{k1}^{20}$ | $R_{k2}^{20}$ | $R_{k3}^{20}$ | $R_{k4}^{20}$ | $R_{k5}^{20}$ | $G_{k}^{20}$ |
|---|---|---|---|---|---|---|---|
| ARM | 1 | 0.99794 | 0.99640 | 0.98876 | 0.98694 | 0.98181 | 0.01819 |
| DDTW | 2 | 0.99385 | 0.98399 | 0.97312 | 0.94591 | 0.85461 | 0.14539 |
| WT | 3 | 1.03360 | 0.98548 | 1.06750 | 0.93302 | 1.07780 | 0.07780 |
| FT | 4 | 1.49200 | 0.81074 | 1.95100 | 0.38305 | 2.02400 | 1.02400 |

Further, based on the $G_k^{10}$ and $G_k^{20}$ (Tables 5, 6), to reflect the so-called "deteriorations" from the application of distance norms $D_k$ in the process of recognizing signals from $S_{10}$ and $S_{20}$ the corresponding coefficients $u_k$ are calculated as follows:

$$u_1 = \frac{G_1^{20}}{G_1^{10}} = 4.8508;$$

$$u_2 = \frac{G_2^{20}}{G_2^{10}} = 9.1154;$$

$$u_3 = \frac{G_3^{20}}{G_3^{10}} = 0.9800;$$

$$u_4 = \frac{G_4^{20}}{G_4^{10}} = 1.8698.$$

The factors $u_k$ are used to identify the weights of the evaluation criteria by follows:

$$\begin{cases} \dfrac{w_k}{w_l} = \dfrac{u_l}{u_k}, \text{ if } k,l = 2,3,4; \\ \sum_{k=1}^{4} w_k = 1. \end{cases} \tag{6}$$

As a result of solving the system of equations (6), the following values were obtained: $w_1$ = 0.11018, $w_2$ = 0.058632, $w_3$ = 0.54535, $w_4$ = 0.28584. Then, the weighted additive convolution formula (5) can be rewritten as follows

$$D = 0.11018 \cdot D_1(x, y) + 0.058632 \cdot D_2(x, y) + \\ + 0.54535 \cdot D_3(x, y) + 0.28584 \cdot D_4(x, y). \tag{7}$$

Further, the convolution (7) was tested on the families $S_h=\{s_0, s_1, s_2, \ldots, s_6\}$, where in each case $s_0$ is the standard, relative to which these families are formed by steps $h$ = 5, 10, 15, 20, 30. For each family the maximum deviations from unity of adjacent distance ratios were obtained and summarized in Table 7.

**Table 7.** Values of maximum deviations from unity of ratios of adjacent distances between reference points of curves from families $S_h$

| Method | $k$ | $G_k^{5}$ | $G_k^{10}$ | $G_k^{15}$ | $G_k^{20}$ | $G_k^{30}$ |
|---|---|---|---|---|---|---|
| ARM | 1 | 0.000699 | 0.003749 | 0.013154 | 0.018188 | 0.25759 |
| DDTW | 2 | 0.003902 | 0.015950 | 0.020242 | 0.145390 | 0.14935 |
| WT | 3 | 0.017309 | 0.079387 | 0.132200 | 0.077800 | 0.46902 |
| FT | 4 | 0.376220 | 0.547660 | 1.043600 | 1.024000 | 2.86460 |
| WAC (7) | | 0.011489 | 0.050017 | 0.098663 | 0.056458 | 0.39412 |

As can be seen from Table 7 and Figure 4, for all families of curves $S_h$ ($h$ = 5, 10, 15, 20, 30), WAC (7) demonstrates smaller deviations from unity than the WT criterion with the highest weight (0.54535).

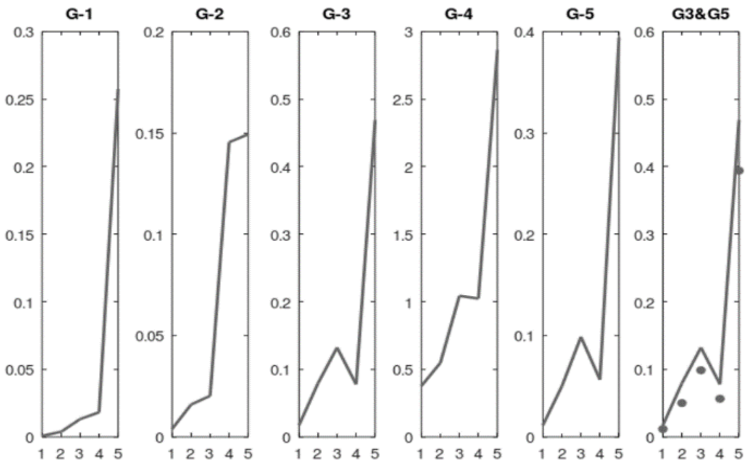


**Fig. 4.** Demonstration of deviations from unity of ratios of adjacent distances between reference points of curves from families $S_h$

## 4. Combining the criteria of the corresponding recognition methods within the additive convolution

In order to simplify the presentation for the considered recognition methods, the following designations are introduced: *A* is the amplitude recognition method; *D* is the method of first derivatives (DDTW); *W* is the Wavelet transform; *F* is the Fourier transform. The essence of combining evaluation criteria by these methods within the additive convolution is to establish the best triple of proximity criteria for joint recognition of signals from a given family. In other words, by combining the composition of the additive convolution by three criteria: *DWF*, *AWF*, *ADF* and *ADW*, it is necessary to determine the best of them in terms of recognition accuracy.

Above, based on the values of the maximum deviations $G_k^{10}$ and $G_k^{20}$ (Tables 5 and 6) for the mentioned four recognition methods, the deterioration coefficients $u_k$ are calculated. In order to establish the corresponding deterioration coefficients for the cases of using additive convolutions *DWF*, *AWF*, *ADF* and *ADW*, the values of the maximum deviations from unity of the ratios of adjacent distances between the points of reference of the curves from the families $S_{10}$ and $S_{20}$ are established by similar actions, which are summarized for each convolution in the following corresponding Tables 8, 9, 10 and 11.

**Table 8**. The values of the maximum deviations $G_k^{10}$ and $G_k^{20}$ from unity of the ratios of adjacent distances between the points of reference of the curves from the $S_{10}$ and $S_{20}$ families using convolution *DWF*

| Recognition method | $k$ | $G_k^{10}$ | $G_k^{20}$ |
|---|---|---|---|
| DDTW | 2 | 0.015950 | 0.14539 |
| WT | 3 | 0.079387 | 0.07780 |
| FT | 4 | 0.547660 | 1.02400 |

**Table 9**. The values of the maximum deviations $G_k^{10}$ and $G_k^{20}$ from unity of the ratios of adjacent distances between the points of reference of the curves from the $s_{10}$ and $s_{20}$ families using convolution *AWF*

| Recognition method | $k$ | $G_k^{10}$ | $G_k^{20}$ |
|---|---|---|---|
| Amplitude | 1 | 0.0039079 | 0.018142 |
| WT | 3 | 0.0793870 | 0.077800 |
| FT | 4 | 0.5476600 | 1.024000 |

**Table 10**. The values of the maximum deviations $G_k^{10}$ and $G_k^{20}$ from unity of the ratios of adjacent distances between the points of reference of the curves from the $S_{10}$ and $s_{20}$ families using convolution *ADF*

| Recognition method | $k$ | $G_k^{10}$ | $G_k^{20}$ |
|---|---|---|---|
| Amplitude | 1 | 0.0039079 | 0.018142 |
| DDTW | 2 | 0.0159500 | 0.145390 |
| FT | 4 | 0.5476600 | 1.024000 |

**Table 11**. The values of the maximum deviations $G_k^{10}$ and $G_k^{20}$ from unity of the ratios of adjacent distances between the points of reference of the curves from the $S_{10}$ and $S_{20}$ families using convolution *ADW*

| Recognition method | $k$ | $G_k^{10}$ | $G_k^{20}$ |
|---|---|---|---|
| Amplitude | 1 | 0.0039079 | 0.018142 |
| DDTW | 2 | 0.0159500 | 0.145390 |
| WT | 3 | 0.0793870 | 0.077800 |

According to the previous considerations, for additive convolutions *DWF*, *AWF*, *ADF* and *ADW*, the deterioration coefficients $u_k$ are calculated by the formula

$$u_k = \frac{G_k^{20}}{G_k^{10}},\ k = 1,2,3.$$

Applying for these cases formula (6) in the form

$$\begin{cases} \dfrac{w_k}{w_n} = \dfrac{u_n}{u_k},\ k,n = 1,2,3, \\ \sum\limits_{k=1}^{3} w_k = 1, \end{cases} \tag{8}$$

the specific weights of evaluation criteria were identified within the additive convolutions *DWF*, *AWF*, *ADF* and *ADW*, which are summarized in Table 12.

**Table 12**. Weights of criteria within the additive convolutions DWF, AWF, ADF AND ADW

| Additive convolution | $w_1$ | $w_2$ | $w_3$ |
|---|---|---|---|
| DWF | 0.06589 | 0.61288 | 0.32123 |
| AWF | 0.12166 | 0.57629 | 0.30205 |
| ADF | 0.25049 | 0.12757 | 0.62194 |
| ADW | 0.16009 | 0.08153 | 0.75837 |

Further, applying each of the considered additive convolutions DWF, ADW, ADF, ADW to recognize signals from the $S_{10}$ family in the form

$$D_{DWF} = 0.06589 \cdot D_D(x, y) + 0.61288 \cdot D_W(x, y) + + 0.32123 \cdot D_F(x, y), \tag{9}$$

$$D_{AWF} = 0.12166 \cdot D_A(x, y) + 0.57629 \cdot D_W(x, y) + + 0.30205 \cdot D_F(x, y), \tag{10}$$

$$D_{ADF} = 0.25049 \cdot D_A(x, y) + 0.12757 \cdot D_D(x, y) + + 0.62194 \cdot D_F(x, y), \tag{11}$$

$$D_{ADW} = 0.16009 \cdot D_A(x, y) + 0.08153 \cdot D_D(x, y) + + 0.75837 \cdot D_W(x, y), \tag{12}$$

and checking them for compliance with Criteria 1 and 2, the ratios of adjacent distances $R_{ki}^{10}$ ($i$=1÷5) and maximum deviations $G_k^{10}$ for the base family of signals $S_{10}$ are identified, which are summarized in Table 13.

**Table 13**. Indicators of fulfillment of criteria 1 and 2 based on the family $S_{10}$.

| Method | $k$ | $R_{k1}^{10}$ | $R_{k2}^{10}$ | $R_{k3}^{10}$ | $R_{k4}^{10}$ | $R_{k5}^{10}$ | $G_k^{10}$ |
|---|---|---|---|---|---|---|---|
| DWF | 1 | 0.97404 | 1.0095 | 1.0742 | 0.98082 | 0.97381 | 0.07422 |
| AWF | 2 | 0.97852 | 1.0076 | 1.0609 | 0.98339 | 0.97831 | 0.06093 |
| ADF | 3 | 1.00210 | 1.0035 | 0.9926 | 0.98751 | 0.99541 | 0.01249 |
| ADW | 4 | 0.98448 | 1.0021 | 1.0350 | 0.99071 | 0.98069 | 0.03497 |

## 5. Discussion of the results

The analysis of the obtained results showed that, despite the small values of the weight coefficients for the first two criteria in the convolution (7), the use of additive convolutions DWF and AWF produces relatively large errors in the form of the corresponding maximum deviations from unity: $G_1^{10} = 0.07422$ and $G_2^{10} = 0.06093$ (Table 13). This is much larger than the error 0.050017 produced by the additive convolution of four criteria (7) (Table 7). It follows from this that even if the weights of the evaluation criteria in the additive convolution are small values, then, excluding them from the convolution, large errors are still observed, since each of the criteria with small specific weights is critical in signal recognition. On the other hand, despite the large values of the weights for the 3rd and 4th criteria in convolution (7), the use of additive convolutions ADF and ADW produce relatively low errors in the form of the corresponding maximum deviations from unity $G_3^{10} = 0.01249$ and $G_4^{10} = 0.03497$ (Table 13), which is much less than the error 0.0500170 produced by the additive convolution of four criteria (7) (Table 7).

From the foregoing, one can conclude that the evaluation criteria of the methods of Wavelet and Fourier transforms "compete" with each other and, thereby, "interfere" with each other. Therefore, their joint use in additive convolution is inappropriate. In particular, excluding the WT criterion from the convolution, that is, when applying the ADF convolution, the smallest error of 0.012486 is achieved. This is explained by the fact that, from the point of view of the recognition accuracy of signals from the given family $S_{10}$, the WT has a lower degree of adequacy than the FT.

The adequacy of the recognition method depends on the correct choice of the feature, according to which, in fact, the comparison of signals is carried out. In our study, for WT indicators of the mean value and root-mean-square distances between the count points of the curves were chosen as a feature extraction method. However, the conducted empirical studies have shown the inadequacy of using the mean value as a feature of comparing curves.

As a mean of extracting features for FT, we have chosen a method based on the use of all the data (features) that represent this recognition method. Despite the fact that the method of applying all the data more adequately reflects the essence of the method, its disadvantage is the excessive redundancy of the data, which cannot always be processed due to the need for large computational effort.

Finally, Table 14 is presented, which shows the results of calculations obtained using additive convolutions DWF, AWF, ADF and ADW to recognize signals from the $S_h$ families ($h$ = 5, 10, 15, 20, 30). Similar results obtained in the process of recognition of signals from other families also confirm the facts revealed in the analysis of the results from Table 13.

**Table 14.** Values of maximum deviations from unity of ratios of adjacent distances between reference points of curves from families $S_h$

| Method | $k$ | $G_k^5$ | $G_k^{10}$ | $G_k^{15}$ | $G_k^{20}$ | $G_k^{30}$ |
|---|---|---|---|---|---|---|
| DWF | 1 | 0.017583 | 0.07422 | 0.13288 | 0.077643 | 0.43867 |
| AWF | 2 | 0.014291 | 0.060928 | 0.11366 | 0.064258 | 0.41463 |
| ADF | 3 | 0.002251 | 0.012486 | 0.026907 | 0.065124 | 0.13218 |
| ADW | 4 | 0.00771 | 0.034965 | 0.079103 | 0.052122 | 0.37715 |

## 6. Conclusion

Combining four well-studied evaluation criteria in the additive convolution made it possible to identify the best combination of the evaluation criteria for recognizing signals from the given family. Such is the additive convolution ADF (formula (11)), which aggregates the recognition results using the amplitude method, the method of first derivatives and the Fourier-Transform.

Within the framework of this study, all experimental calculations were carried out using programs written in high-level algorithmic languages such as Octave and Python.

## References


Al-Qerem, A., Kharbat, F. Nashwan, Sh., Ashra, S. (2020). General model for best feature extraction of EEG using discrete wavelet transform wavelet family and differential evolution. International Journal of Distributed Sensor Networks 16(3). https://doi.org/10.1177/1550147720911009

Cedro, L., Janecki, D. (2011). Determining of signal derivatives in identification problems, FIR Differential Filters, Acta Montanistica Slovaca Rocník 16(1), 47-54.

Hindarto, H., Anshory, I., Efiyanti, A. (2017). Feature extraction of heart signals using fast Fourier transform, UNEJ e-Proceeding 165-167. https://jurnal.unej.ac.id/index.php/prosiding/article/view/4187

Keogh, E.J., Pazzani, M.J. Derivative Dynamic Time Warping. https://epubs.siam.org/doi/epdf/10.1137/1.9781611972719.1

Kerimov, A.B. (2022, a). Accuracy comparison of signal recognition methods on the example of a family of successively horizontally displaced curves, Informatics and Control Problems 42(2), 80-91.

Kerimov, A.B. (2022, b).Comparison of some signal recognition methods for their adequacy, Proceedings of the 8th International Conference on Control and optimization with industrial applications, Vol. I.

Leszek, C., Janecki, D., Determining of signal derivatives in identification problems, FIR Differential Filters, Acta Montanistica Slovaca Rocník 16(1), 47-54 (2011).

Liu, Y., Lin, J. (2019). A general-purpose signal processing algorithm for biological profiles using only first-order derivative information. BMC Bioinformatics 20, 611 https://doi.org/10.1186/s12859-019-3188-4

Novozhilov, B.M. (2016). Calculation of the derivative of an analog signal in a programmable logic controller. Aerospace Scientific Journal of the Moscow State Technical University N.E. Bauman, Electron Journal 4, 1-12 (In Russian).

Ponomarev, S., Wallace, N., Atkison, T. Fourier transform as feature extraction for malware classification. http://atkison.cs.ua.edu/papers/FT_as_FE.pdf

Santos, M., Morais, C., Nascimento, Y., Araujo, J., Lima, K. (2017). Spectroscopy with computational analysis in biological studies: a decade (2006–2016). TrAC Trends Anal Chem 97, 244-256

Saraswat, S., Srivastava, G., Sachchidanand, N. (2017). Wavelet transform based feature extraction and classification of atrial fibrillation arrhythmia. Biomed Pharmacol J 10(4) http://biomedpharmajournal.org/?p=17470

Song T., Yu X., Yu S., Ren Z., and Qu Y. (2021). Feature extraction processing method of medical image fusion based on neural network algorithm, Hindawi Complexity https://doi.org/10.1155/2021/7523513

Taghavirashidizadeh, A., Sharifi, F., Vahabi, S.A., Hejazi, A., Torbati, M.S., Mohammed, A.S. (2022). WTD-PSD: presentation of novel feature extraction method based on discrete Wavelet transformation and tme-dependent power spectrum descriptors for diagnosis of Alzheimer's disease, Computational Intelligence and Neuroscience https://doi.org/10.1155/2022/9554768