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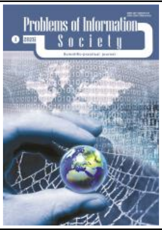

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Modelling of torque-free rotational dynamics along principal axes of a spacecraft

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ABSTRACT

The rotational behavior of spacecraft plays a critical role in the stability and control of satellite missions. In a torque-free environment, this behavior is governed by Euler's equations, which describe how angular velocity evolves based on the spacecraft's inertia properties. This study investigates the stability of spacecraft rotation about each principal axis, focusing on the influence of mass distribution and axis selection. The aim is to identify how different configurations affect rotational stability, especially near the intermediate axis where motion can become unpredictable. A simulation framework was developed using programming languages C for numerical integration and Python for visualization and analysis. The study reveals that rotation about the major and minor axes remains stable, while rotation about the intermediate axis leads to unstable behavior, confirming theoretical expectations. Angular velocities and attitude parameters were computed and visualized to illustrate these dynamics. The findings offer valuable insight into passive attitude dynamics and are relevant for designing robust attitude control strategies in space missions.

1. Introduction

The motion of a spacecraft consists of two main components: translation and rotation. While translational motion determines the orbit of the spacecraft, rotational motion governs its orientation in space. The rotational dynamics of a rigid spacecraft are described by Euler's equations, which relate the angular velocity of the body to its moments of inertia and angular momentum (Hughes, 1986). In many practical scenarios, especially during safe-mode operations or initial deployment, the spacecraft may undergo torque-free rotation. In such cases, the internal dynamics of the body alone determine whether the motion remains stable or becomes unpredictable (Landau & Lifshitz, 1976; Wie, 2008).

The way a spacecraft rotates depends heavily on how its mass is distributed relative to its principal axes. These axes, known as the major, intermediate, and minor axes. Each of these principal axes has different stability characteristics. A well-known behavior in this context is the instability that arises when a spacecraft rotates its intermediate axis. This phenomenon, sometimes referred to as the Dzhanibekov effect (Van Damme et al., 2016), causes the rotational motion to become unstable even in the absence of external forces. In contrast, rotation about the major or minor axis generally remains stable. This dynamic instability has been observed in both theoretical studies and real-life demonstrations (Black, 1964), and it poses significant challenges in spacecraft design, especially when relying on passive attitude stabilization (Wertz, 1978).

Although the theory behind principal axis rotation is well established, it is still important to study these dynamics numerically. Many satellites today are designed for autonomous operations with limited control authority and minimal ground intervention. Understanding how the spacecraft behaves under torque-free conditions can help improve the reliability of these missions (Hughes, 1986). In particular, simulating the motion along different axes provides engineers with insight into which spin configurations are safest and most predictable for specific mission phases.

The outcomes of this study support the design of robust attitude control strategies and contribute to a better understanding of the internal dynamics of spacecraft in uncontrolled or minimally controlled states.

2. Related work

The study of torque-free spacecraft dynamics has been revisited in recent years, with several works extending classical rigid-body theory into new applications. One important direction has been the role of inertia properties in determining stability. Reorientation of spinning bodies was shown to be achievable by deliberately evolving the inertia tensor, which highlights the strong dependence of stability on mass distribution (Ostanin & Sperl, 2024). Similar approaches that combine modeling and simulation have emphasized controllability along principal axes, showing how torque-free dynamics set natural limits on feasible spacecraft maneuvers (Fiori et al., 2024).

Beyond reorientation strategies, considerable attention has been given to the estimation of inertia parameters for uncooperative objects that often rotate in near torque-free conditions. Experimental studies have confirmed that inertia and attitude estimation can be performed effectively in laboratory environments designed to mimic on-orbit conditions (Nocerino et al., 2023). More advanced techniques have also been developed, including a two-stage estimator that can recover the complete inertia tensor of debris objects (Creaser & Bauer, 2024) and Unscented Kalman Filter-based approaches for estimating inertia, angular velocity, and attitude of tethered debris (Bourabah et al., 2023).

Observational studies provide an additional perspective by analyzing natural spin states of space objects. For example, recent work has applied photometric light-curve data to classify the attitude motion of resident space objects, showing how tumbling debris can be distinguished from more stable spin modes (Isoletta et al., 2025). These works reinforce that torque-free or weakly perturbed dynamics are not only of theoretical interest but also of practical importance in debris characterization and mission planning.

Taken together, prior research covers theoretical analysis, reorientation control, parameter estimation, and observational validation. However, fewer studies have presented a systematic comparison of torque-free stability across all three principal axes using a representative inertia tensor and direct simulation of angular velocity and quaternion evolution.

The present study addresses this gap by providing a focused numerical investigation of axis-dependent spin behavior, contributing a baseline reference for spacecraft designers and mission planners.

3. Material and methods

In this section, the approach used to study the torque-free rotational motion of the spacecraft is explained. First, the problem is defined in terms of how rotation depends on the mass distribution and the choice of principal axis. Then, the simulation setup and initial conditions are described, followed by the results of numerical runs for each axis. Together, these steps show how the theoretical background was applied and tested in practice.

3.1. Problem Statement

The rotational behavior of spacecraft in a torque-free environment is governed by its mass distribution and the axis about which it spins. While theoretical dynamics are well understood through

Euler's equations and the intermediate axis theorem (Hughes, 1986; Van Damme et. al, 2016) practical analysis often requires numerical simulation to observe how these dynamics manifest under realistic initial conditions (Kane, Likins, & Levinson, 1983). Spin stability differs significantly depending on whether rotation occurs about the major, minor, or intermediate principal axis (Van Damme et al., 2016).

Despite the known theory, there is a lack of focused studies that clearly demonstrate the differences in stability behavior across all three principal axes using representative spacecraft inertia and angular velocity profiles. Without such simulation-based insight, engineers may overlook important dynamic effects that influence control design, mission safety, or passive stabilization strategies (Wie, 2008).

This study addresses this gap by performing a numerical analysis of spin stability for each principal axis using torque-free conditions and a realistic inertia tensor. The results aim to clarify how mass distribution and axis selection affect long-term rotational behavior with implications for both spacecraft design and control system planning.

3.2. Problem solution

To simulate the spacecraft behavior, the free rotational motion of a rigid spacecraft is analyzed using numerical simulation. Euler's equations are integrated using a custom-built tool developed in the C programming language to ensure computational efficiency. The equations of motion governing the torque-free rotational dynamics of the spacecraft which is called Euler's equation as well expressed as follows.

$$I\dot{\omega} + [\omega] \times (I\omega) = M \qquad (1)$$

Where $\boldsymbol{I}$ is the 3x3 inertia matrix, $\boldsymbol{\omega}$ is 3x1 angular velocity vector and $\boldsymbol{M}$ is the 3x1 external torques vector acting on the spacecraft.

When the inertia matrix is diagonalized with aligning the body axes with the principal axes of inertia, only the diagonal elements of $\boldsymbol{I}$ remain nonzero. Consequently, under this assumption, equation (1) reduces to the scalar form which significantly reduces computational cost as well as making it well suited for numerical simulations. The scalar form of Euler's equation can be described as

$$\begin{aligned} I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 &= M_1 \\ I_2\dot{\omega}_2 - (I_3 - I_1)\omega_3\omega_1 &= M_2 \\ I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 &= M_3 \end{aligned} \qquad (2)$$

Throughout the analysis, external torques are assumed zero. This torque-free condition aligns with the study's goal of observing natural attitude behavior (Wie, 2008; Wertz, 1978). While external torques such as gravity gradient or magnetic disturbances can significantly influence spacecraft dynamics in operational scenarios, they are excluded here to focus purely on the internal effects of mass distribution and initial spin alignment (Thomson, 1986). Consequently, the final equation of motion, in other words, Euler's equations for each of the angular velocity component become as follows:

$$\begin{aligned} \dot{\omega}_1 &= \frac{(I_2 - I_3)\omega_2\omega_3}{I_1} \\ \dot{\omega}_2 &= \frac{(I_3 - I_1)\omega_3\omega_1}{I_2} \\ \dot{\omega}_3 &= \frac{(I_1 - I_2)\omega_1\omega_2}{I_3} \end{aligned} \qquad (3)$$

After integrating the angular velocity components using equation (3) in C programming language and exporting the results to file as Comma-Separated Values (CSV) format, the simulation data are visualized and analyzed in Python. Several initial conditions are considered, each corresponding to spin about one of the three principal axes. The evolution of angular velocity and attitude parameters is observed and compared to assess the stability of each case.

The simulation workflow outlining the main steps implemented in C and Python is shown in fig. 1.

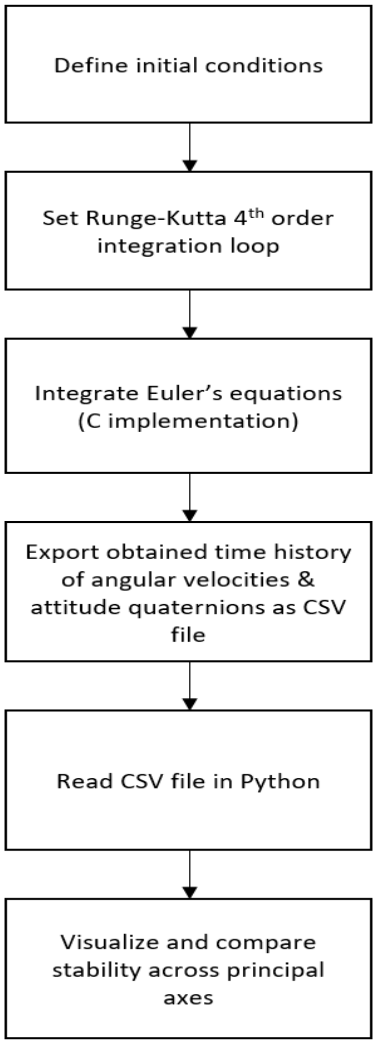

Fig. 1. Simulation workflow for torque-free rotational dynamics

3.3. Initial conditions

A specific set of initial conditions and an appropriate inertia tensor were defined. The selected inertia matrix is given by:

$$I = \begin{bmatrix} 150 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

As you can see from the matrix, this configuration represents a rigid spacecraft with an asymmetric mass distribution where the moment of inertia about the X-axis is the largest, the Y-axis is intermediate, and the Z-axis is the smallest (Sidi, 1967). This selection is intentional, as it directly influences the rotational dynamics experienced when the spacecraft spins about each of its principal axes. If all diagonal elements of the inertia matrix were equal, the body would exhibit spherical symmetry. In that case, the rotational behavior would be identical for all axes. However, with the defined matrix, the system exhibits distinct dynamics depending on the axis of rotation (Vallado, 2013).

In addition to the inertia properties, the initial angular velocity values are critical in determining the nature of the motion. For each case study, the spacecraft is assumed to rotate primarily about a single principal axis. To reflect this, the *angular* velocity component corresponding to the axis under analysis is assigned a relatively large value, while the components along the other two axes are set to small perturbations (Hughes, 1986; Prussing & Conway, 2013). This ensures that the simulation isolates the behavior of interest while still allowing potential instability to develop.

The simulation was done for three principal axes cases and following initial angular velocity vectors used for each cases:

- Case 1 (rotation about X-axis)
  $\omega_0 = [10, 0.5, 0.5]$
- Case 2 (rotation about Y-axis)
  $\omega_0 = [0.5, 10, 0.5]$
- Case 3 (rotation about Z-axis)
  $\omega_0 = [0.5, 0.5, 10]$

3.4. Numerical experiments and results

In this subsection, the outcomes of numerical simulations of torque-free rotation motion along the principal axes of the spacecraft are presented. Each axis is analyzed separately in terms of angular velocity, attitude evolution and stability characteristics.

3.4.1. Rotation about the X-axis

In the first case, the initial angular velocity was aligned with the spacecraft's principal X-axis. This case is considered to represent rotation about the stable axis of inertia. The results are presented in terms of time evolution of angular velocity shown in fig. 2 and the variation of quaternion components over time shown in fig. 3.

As observed in the angular velocity plot for rotation about the X-axis in fig. 2, the spacecraft maintains a stable spin with the first component remaining nearly constant around 10 deg/s. This behavior is expected because the initial angular velocity is aligned with the principal axis that also has the highest moment of inertia.

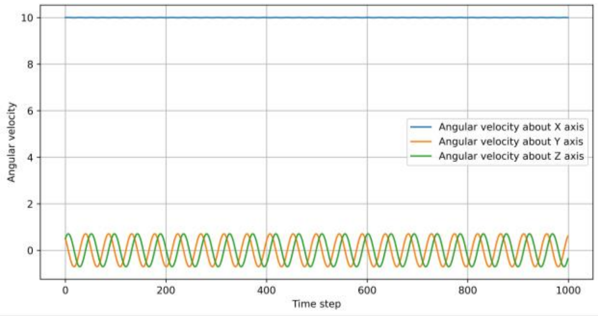

Fig. 2. Time evolution of angular velocity about first principal axis

The alignment of high rotational energy with the most dominant mass axis results in minimal interaction with the other axes, which results in stability of motion.

Meanwhile, the angular velocity components of the Y and Z-axis exhibit small and periodic oscillations. These arise from the slight initial perturbations applied to those directions but remain bounded throughout the simulation.

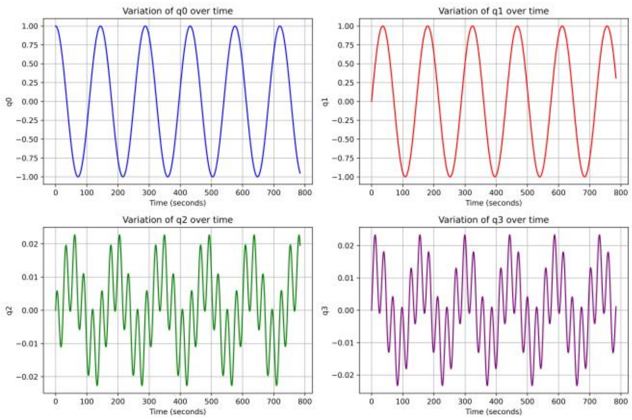

Fig. 3. Variation of quaternion components over time about first principal axis

The quaternion components in Fig. 3 show behavior consistent with expectations for rotation about the X-axis. The component q_1, which corresponds to the rotational alignment along the X-axis, exhibits a large-amplitude sinusoidal pattern. As the initial angular velocity is dominant along the X-axis, and q_1 reflects that rotational direction, this behavior is expected.

In contrast, the q_2 and q_3 components, which correspond to the Y and Z-axis respectively, remain close to zero and oscillate with very small amplitudes, not exceeding 0.02. These minor fluctuations are due to the small initial angular velocity components applied in the Y and Z-axis directions.

The scalar component q_0 where represents the overall rotation magnitude and phase in the quaternion formulation, also shows smooth, large-amplitude oscillation. This indicates that the spacecraft maintains a consistent rotational state throughout the simulation, which confirms the expected stable behavior when spinning the X-axis under torque-free conditions.

3.4.2. Rotation about the Y-axis

In the second case, the initial angular velocity was aligned along the principal Y-axis which is the intermediate axis of inertia. This axis is known to be dynamically unstable, a phenomenon often referred to as the tennis racket theorem or Dzhanibekov effect. The results are shown through time evolution of angular velocity in fig. 4 and the variation of quaternion components over time in fig. 5.

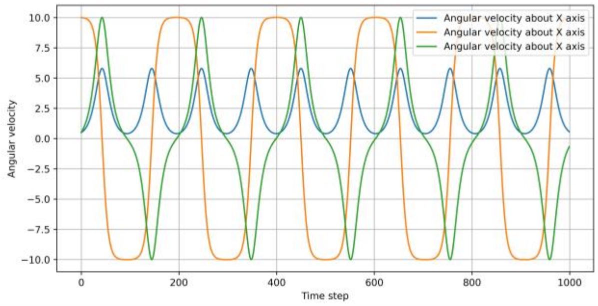

Fig. 4. Time evolution of angular velocity about second principal axis

The angular velocity plot corresponding to rotation about the Y-axis in fig. 4, reveals irregular and seemingly chaotic motion across all three components including the dominant ω_y direction. This behavior is characteristic of separatrix motion, a complex dynamical regime where the spacecraft's rotation exhibits sensitive dependence on initial conditions and unpredictable transitions between rotational states. In this regime, the angular velocity does not remain confined to a single axis but fluctuates non-periodically. Consequently it leads to uneven and sometimes recurring shifts in the direction and magnitude of the spin.

Such motion can result in inconsistent orientation changes during free rotation and complicate predictions of the spacecraft's attitude over time. The dynamics are highly nonlinear and cannot be captured by simple sinusoidal models, underscores the sensitivity and instability inherent to rotation near the intermediate axis.

The quaternion components in fig. 5 exhibit irregular patterns that are characteristic of separatrix conditions.

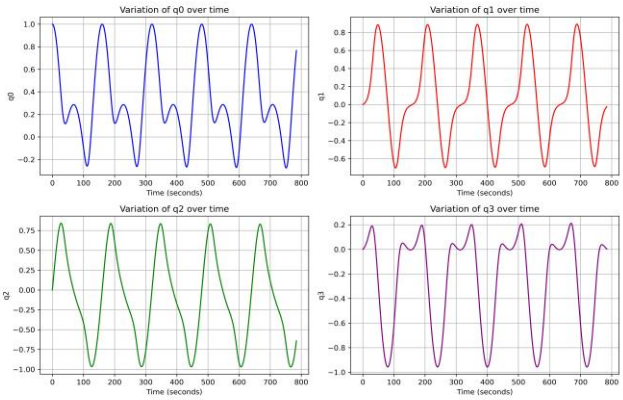

Fig. 5. Variation of quaternion components over time about second principal axis

As shown in the plots, none of the quaternion elements follows a stable oscillatory pattern. Instead, their trajectories appear disordered and lack any clearly identifiable waveforms. This reflects the system's sensitivity to small perturbations and initial conditions which is a defining feature of separatrix dynamics.

Although the quaternion representation is continuous and free of singularities, it can still give the illusion of linearity or smoothness in certain segments of the trajectory. This may lead to misinterpretation of the spacecraft's true rotational behavior if the underlying instability is not considered. In reality, the system transitions between multiple regions of the phase space in a non-deterministic manner which makes the long-term prediction of attitude evolution inherently unreliable in this regime.

3.4.3. Rotation about the Z-axis

In the third case, the initial angular velocity was aligned with the spacecraft's principal Z-axis. Although differences are observed in angular velocity and attitude evolution, this axis, similar to the X-axis case, is theoretically considered dynamically stable. The corresponding results are given by the time evolution of angular velocity plot in fig. 6 and the variation of quaternion components over time in fig. 7.

During rotation about the Z-axis, the angular velocity components in fig. 6, display a regular and predictable pattern without any signs of chaotic behavior. The dominant angular velocity of third axis remains close to its initial value of 10 degree/s. This consistent behavior confirms the stability of rotation about the minor principal axis when subjected to torque-free conditions.

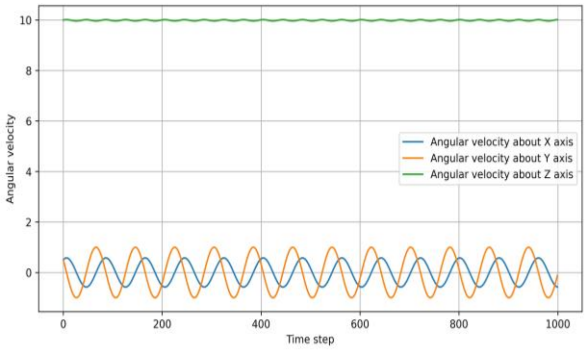

Fig. 6. Time evolution of angular velocity about third principal axis

The X and Y-axis components of angular velocity exhibit harmonic oscillations with small amplitudes, which are expected due to their initial perturbations. These minor variations are synchronized with the dominant Z-axis motion and do not indicate any instability. Small fluctuations around the angular velocity of third axis are also observed, which is consistent with the behavior of a dynamically stable system attempting to maintain its rotational state through numerical integration.

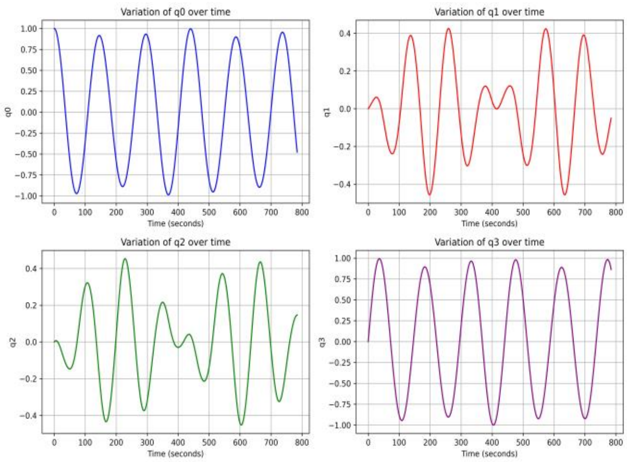

Fig. 7. Variation of quaternion components over time about third principal axis

The quaternion results in fig. 7 further confirm the stability of the motion. Since the rotation is primarily about the Z-axis, both the scalar component q_0 and the third vector component q_3 exhibit large amplitude sinusoidal variations. These components are directly associated with the dominant axis of rotation and reflect the smooth and periodic nature of the spacecraft's orientation.

In contrast, the q_1 and q_2 components remain small and oscillate in a manner that adapts to the dominant Z-axis motion. Their low amplitudes indicate minimal orientation influence along the X and Y-axis, aligning with the expected behavior of a stable spin around the axis with the smallest moment of inertia.

The summarized numerical outcomes for angular velocity and quaternion components are presented in Tables 1 and 2, respectively, for comparison across all principal axes.

Table 1. Summary of angular velocity behavior under torque-free rotation

Principal axis	Dominant Component	Mean Angular Velocity	Stability behavior
X (major)	ω_x	≈10.0	Stable – bounded oscillations
Y (intermediate)	ω_y	Irregular, time-varying	Unstable – chaotic motion
Z (minor)	ω_z	≈10.0	Stable – periodic oscillations

Table 2. Summary of quaternion behavior under torque-free rotation

Principal axis	Dominant Component	Behavior Type	Stability
X (major)	q_1	Sinusoidal, bounded	Stable
Y (intermediate)	q_2	Irregular, chaotic	Unstable
Z (minor)	q_3	Smooth, periodic	Stable

4. Discussion

Simulation results align with predictions from classical rigid body dynamics. Rotation around the X-axis (major principal axis with largest moment of inertia) demonstrated stable behavior, with the angular velocity component maintaining approximately constant value and the quaternion components undergoing smooth bounded oscillations. Rotation around the Z-axis (minor principal axis with smallest moment of inertia) also showed stable spin with bounded harmonic oscillations of off-axis angular velocity and quaternion components. These simulations confirm that spin about the major or minor principal axes results in a stable rotation.

On the other hand, rotation around the Y-axis (intermediate principal axis) exhibited dynamically unstable behavior. The components of angular velocity and the quaternion both showed irregular fluctuations. This behavior is consistent with known separatrix behavior and is marked by erratic and chaotic attitude transitions due to sensitivity to initial conditions.

In addition to validating theoretical predictions, the simulations provide a quantitative sense of the time evolution of these instabilities. The stark contrast between smooth oscillatory behavior for rotations about the stable axes (X and Z) and the erratic fluctuations about the intermediate axis (Y) underscores the practical implications for passive spin stabilization and the critical importance of selecting an appropriate spin axis, a consideration that is particularly important for small satellites and CubeSats which often have simple design parameters and limited control authority.

Additionally, the numerical simulation results highlight the impact that even small deviations in the inertia tensor (such as minor asymmetries) or small perturbations in the initial angular velocity can have on the evolution of an intermediate-axis rotation. This sensitivity to initial conditions helps to explain why uncontrolled spin along the intermediate axis may occur in real missions, whereas the robustness of the X- and Z-axis rotations explains their prevalence as the default choice for safe-mode or momentum-biased stabilization strategies.

Future analysis could extend the current results to account for the influence of external torques, environmental disturbances, or active control responses to further validate and compare with the results presented in this work.

For instance, a possible future extension could be to compare results from purely numerical simulations to experimental data obtained from laboratory testbeds or previously flown spacecraft. This could help validate the findings and bring more practical relevance to the purely theoretical dynamical results. Another extension could be to analyze how changes in inertia properties, such as fuel depletion or deployment of appendages, affect the stability margins derived in this analysis.

5. Conclusion

The presented results demonstrated a torque-free numerical analysis of spacecraft spin motion about each of the principal axes. Given a specific inertia tensor and a unique initial angular velocity vector in each case, the time responses of the angular velocity and quaternion were analyzed to determine the dynamic behavior of each spin mode.

In general, the results of the torque-free analysis reiterate the significance of axis selection and mass distribution on the passive behavior of a spacecraft's attitude dynamics. Such findings can be of use to spacecraft designers and mission planners who wish to take advantage of free rotation or momentum-based methods of stabilization.

The present work lays a foundation for more comprehensive studies on spacecraft attitude dynamics and reinforces the continuing importance of torque-free rotational analysis in both academic research and engineering applications.

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